## **MM2551**

# Study program / course: Mechanical engineering Type and level of study: Master academic studies

Course: Nonlinear analysis
Lecturers: Miroslav M. Zivkovic

Status of course: Obligatory for module M<sub>5</sub>, II semester

**Number of ECTS: 6** 

# **Precondition:**

## The objective of course

Understanding theoretical basics of nonlinear continuum mechanics and its application in nonlinear analysis of structures using finite elements method. Introduction to a static and dynamic nonlinear FEM analysis. Application of FEM in nonlinear analysis of real engineering problems.

#### The outcome of course

Students will, upon passing exam Advanced analysis and computer system simulation:

know theoretical basics of nonlinear continuum mechanics;

understand basics of modeling and static and dynamic nonlinear analysis using finite element methods;

know to apply gained knowledge on modeling and nonlinear analysis of real engineering problems.

### **Syllabus**

**Theoretical study:** Introduction to the nonlinear analysis of structures. Concept of geometrical and material nonlinearity. Fundamentals of nonlinear continuum mechanics. Lagrange and Eulerian description of motion. Reference and current configuration. Strain gradient, polar decomposition. Strain measures for large strain: left and right Cauchy deformation tensor, Green-Lagrange, Almansi strain tensor. Generalized strain measures, logarithmic strain. Velocity gradient and strain rate. Energy conjugate stress-strain measures; Cauchy, 2<sup>nd</sup> Piola-Kirchhoff stress tensor. Constitutive relations. *Linearization of equation of motion:* Principe of virtual work and differential equation of motion. Total and update Lagrange formulation. Linearization of equation of motion, linear and geometrical nonlinear stiffness matrix, mass matrix and internal forces vector. Incremental-iterative equilibrium equation. Solution methods for nonlinear equations. Newton and modified Newton methods. Convergence criterions. *Material nonlinearity:* Integration of constitutive relations in incremental iterative solution procedure of displacement method. Isotropic metal plasticity and governing parameter method. *Finite element matrix creation:* Solid elements: 2-D and 3-D; structural elements: shell and beam elements. Definitions of geometrically nonlinear stiffness matrix for total and update Lagrangian formulation. Improving finite elements using incompatible modes. *Nonlinear dynamic analysis:* Explicit integration. Implicit integration.

#### **Practical Studies:**

Determination of deformation gradient from given displacement field, using Jacobi matrix. Left and right Cauchy deformation tensor calculation. Principal directions and principal values of strain tensor. Calculation of Green-Lagrange and Almanasi strain tensors. Transformation of Cauchy stress tensor in 2<sup>nd</sup> Piola-Kirchhoff stress tensor. Simple examples of geometrical nonlinearity (total and update Lagrange formulation). Simple examples of isotropic metal plasticity. Examples for solving complex problems, in geometrical and material nonlinear analysis constructions, with program package PAK, using enhanced finite elements for nonlinear analysis: 2-D, 3-D, shell and beam.

## **Recommended reading**

Š. Dunica, B. Kolundžija: Nonlinear analysis of structures, Faculty of Civil Engineering, Beograd, 1986.

M. Živković: Nonlinear analysis of structures, Faculty of Mechanical Engineering, Kragujevac, 2006.

The number of hou	Other classes: 1			
Theory: 2	Practical classes:	Other forms of	Research study: 0	
	1.6	teaching: 0.4		

# Methods of teaching

Teaching is conducted through lectures, practical classes and independent work of students. Within lectures student receives basic theoretical knowledge. In practical classes students receive practical knowledge and skill for using CAD and FEM tools. Students create independent tasks which include and integrate knowledge for usage of certain tools.

Evaluation of knowledge					
Pre-final exam	points	Final exam	points		
obligations					
Activities during the	5		30		
classes:					
Practical classes/	30				
Home works:					
Colloquiums(s)/ Tests:	35				
Seminar(s):					